22147305

International Baccalaureate®
Baccalauréat International
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MATHEMATICS
STANDARD LEVEL
PAPER 1

Candidate session number

Tuesday 13 May 2014 (afternoon)

1 hour 30 minutes

Examination code

2	2	1	4	–	7	3	0	5

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

Please **do not** write on this page.

Answers written on this page
will not be marked.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 5]*

The following diagram shows a right-angled triangle, ABC, where $\sin A = \frac{5}{13}$.

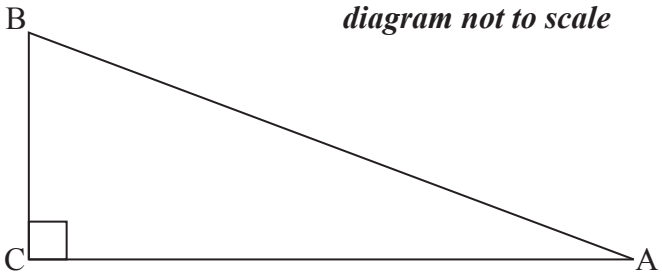

diagram not to scale

(a) Show that $\cos A = \frac{12}{13}$. *[2]*

(b) Find $\cos 2A$. *[3]*

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2. *[Maximum mark: 7]*

Find the value of each of the following, giving your answer as an integer.

(a) $\log_6 36$ *[2]*

(b) $\log_6 4 + \log_6 9$ *[2]*

(c) $\log_6 2 - \log_6 12$ *[3]*

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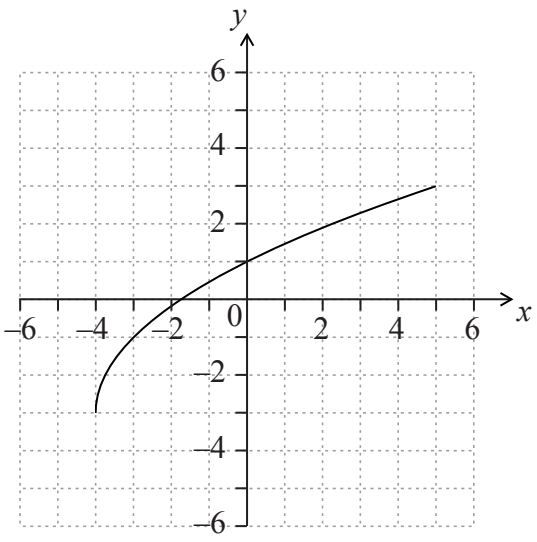
3. *[Maximum mark: 6]*

The following diagram shows the graph of $y = f(x)$, for $-4 \le x \le 5$.



(a) Write down the value of

(i) $f(-3)$;

(ii) $f^{-1}(1)$. *[2]*

(b) Find the domain of f^{-1}. *[2]*

(c) On the grid above, sketch the graph of f^{-1}. *[2]*

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4. *[Maximum mark: 7]*

The line L is parallel to the vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

(a) Find the gradient of the line L. *[2]*

The line L passes through the point $(9, 4)$.

(b) Find the equation of the line L in the form $y = ax + b$. *[3]*

(c) Write down a vector equation for the line L. *[2]*

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5. *[Maximum mark: 6]*

The graph of a function h passes through the point $\left(\frac{\pi}{12}, 5\right)$.

Given that $h'(x) = 4\cos 2x$, find $h(x)$.

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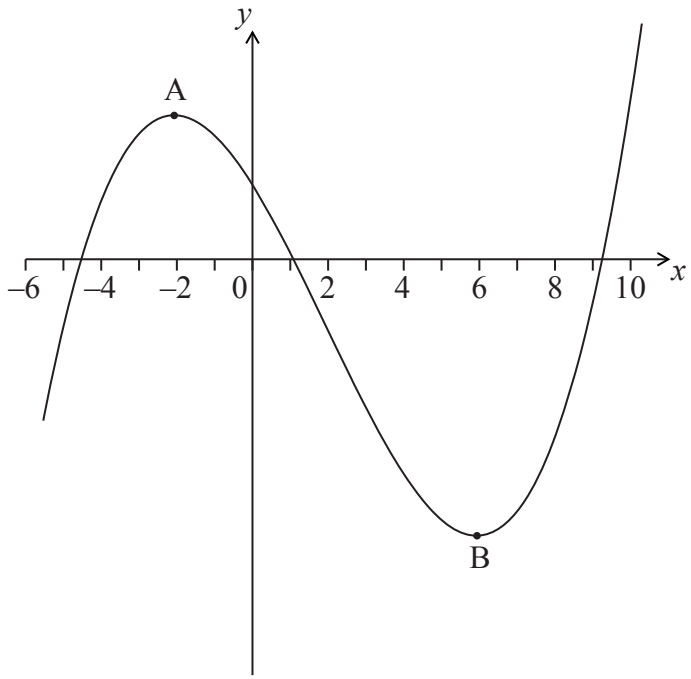
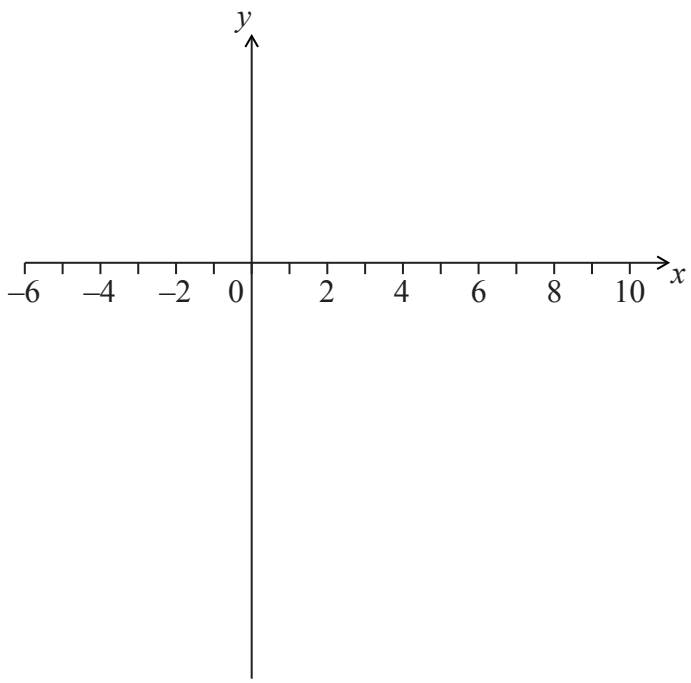
6. *[Maximum mark: 6]*

The following diagram shows part of the graph of $y = f(x)$.



The graph has a local maximum at A, where $x = -2$, and a local minimum at B, where $x = 6$.

(a) On the following axes, sketch the graph of $y = f'(x)$. *[4]*



(This question continues on the following page)

(Question 6 continued)

(b) Write down the following in order from least to greatest: $f(0)$, $f'(6)$, $f''(-2)$. *[2]*

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7. *[Maximum mark: 8]*

The sums of the terms of a sequence follow the pattern

$S_1 = 1 + k$, $S_2 = 5 + 3k$, $S_3 = 12 + 7k$, $S_4 = 22 + 15k$, …, where $k \in \mathbb{Z}$.

(a) Given that $u_1 = 1 + k$, find u_2, u_3 and u_4. *[4]*

(b) Find a general expression for u_n. *[4]*

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*Do **NOT** write solutions on this page.*

SECTION B

*Answer **all** questions in the answer booklet provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

Let $f(x) = 3x^2 - 6x + p$. The equation $f(x) = 0$ has two equal roots.

(a) (i) Write down the **value** of the discriminant.

(ii) Hence, show that $p = 3$. *[3]*

The graph of f has its vertex on the x-axis.

(b) Find the coordinates of the vertex of the graph of f. *[4]*

(c) Write down the solution of $f(x) = 0$. *[1]*

(d) The function can be written in the form $f(x) = a(x-h)^2 + k$. Write down the value of

(i) a;

(ii) h;

(iii) k. *[3]*

(e) The graph of a function g is obtained from the graph of f by a reflection of f in the x-axis, followed by a translation by the vector $\begin{pmatrix} 0 \\ 6 \end{pmatrix}$. Find g, giving your answer in the form $g(x) = Ax^2 + Bx + C$. *[4]*

*Do **NOT** write solutions on this page.*

9. *[Maximum mark: 15]*

Distances in this question are in metres.

Ryan and Jack have model airplanes, which take off from level ground. Jack's airplane takes off after Ryan's.

The position of Ryan's airplane t seconds after it takes off is given by $\boldsymbol{r} = \begin{pmatrix} 5 \\ 6 \\ 0 \end{pmatrix} + t \begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$.

(a) Find the speed of Ryan's airplane. *[3]*

(b) Find the height of Ryan's airplane after two seconds. *[2]*

The position of Jack's airplane s seconds after **it** takes off is given by $\boldsymbol{r} = \begin{pmatrix} -39 \\ 44 \\ 0 \end{pmatrix} + s \begin{pmatrix} 4 \\ -6 \\ 7 \end{pmatrix}$.

(c) Show that the paths of the airplanes are perpendicular. *[5]*

The two airplanes collide at the point $(-23, 20, 28)$.

(d) How long after Ryan's airplane takes off does Jack's airplane take off? *[5]*

*Do **NOT** write solutions on this page.*

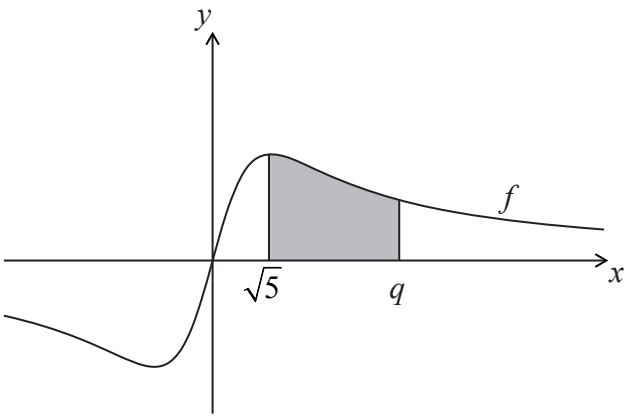
10. *[Maximum mark: 15]*

Let $f(x) = \frac{2x}{x^2+5}$.

(a) Use the quotient rule to show that $f'(x) = \frac{10-2x^2}{(x^2+5)^2}$. *[4]*

(b) Find $\int \frac{2x}{x^2+5}\mathrm{d}x$. *[4]*

The following diagram shows part of the graph of f.



(c) The shaded region is enclosed by the graph of f, the x-axis, and the lines $x = \sqrt{5}$ and $x = q$. This region has an area of $\ln 7$. Find the value of q. *[7]*

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